The data for $0 < \mu < 1$ are inconclusive but seem to indicate that the rotation of the outer cylinder stabilizes the flow against the formation of wavy vortices.

Observations of the flow patterns between the first and second critical points (for $\mu = 0$) were also made, and the appearance of a transition region between steady square Taylor vortices and wavy vortices was noticed.

ACKNOWLEDGMENT

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NOTATION

d = gap width
Fg = correction factor for finite gap widths
n = speed of rotation of cylinders (rev./min.)
P = stability parameter based on δ
r = radius
r_0 = radius of zero velocity streamline
Ta = Taylor number
$\overline{V}$ = average velocity of flow in potentially unstable region of annulus
Z = symbol used in defining Fg
v = velocity
δ = potentially unstable region of gap based on Couette flow equation
μ = ratio of cylinder speeds (ω_1/ω_2); negative for rotation in opposite directions
ν = kinematic viscosity
ω = speed of rotation of cylinders (radian/sec.)

Subscripts

1 = inner cylinder
2 = outer cylinder
c = critical point for steady vortices
w = critical point for wavy vortices
m = average value, for example $r_m = \dfrac{r_1 + r_2}{2}$

LITERATURE CITED

1. Taylor, G. I., *Phil. Trans. Roy. Soc. London,* **A223**, 289 (1923).
2. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability," The Clarendon Press, Oxford, England (1961).
3. Kaye, J., and E. C. Elgar, *Trans. Am. Soc. Mech. Engrs.,* **80**, 753 (1958).
4. Chandrasekhar, S., *Mathematika,* **1**, 5 (1954).
5. Di Prima, R. C., *Quart. App. Math.* **13**, 55 (1955).
6. Brewster, D. B., P. Grosberg, and A. H. Nissan, *Proc. Roy. Soc. (London),* **A251**, 76 (1959).
7. Schultz-Grunow, F., and H. Hein, *Forschungsber. Landes Nordrhein-Westfalen, No. 684* (1959).
8. Coles, D., Paper presented at the Tenth Intern. Congr. of Appl. Mech., Stress, Italy (August, 1960).
9. Di Prima, R. C., *Phys. Fluids,* **4**, 751 (1961).
10. Appel, D. W., *Tappi,* **42**, 764 (1959).
11. Lewis, J. W., *Proc. Roy. Soc. (London),* **A117**, 388 (1927).
12. Krueger, E. R., and R. C. Di Prima, *Math. Report No. 52,* Rensselaer Polytechnic Institute, Troy, New York (March, 1962).
13. Bisshopp, F. E., Office Naval Res., *Tech. Rept. 48,* (1962).
14. Krueger, E. R., Ph.D. thesis, Rensselaer Polytechnic Institute, Troy, New York (1962)

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Experimental Study of Laminar Flow Heat Transfer with Prescribed Wall Heat Flux

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Laminar-flow heat transfer in a cylindrical tube was studied experimentally in order to test a recent analytical method of predicting temperature profiles (1). The experimental equipment was designed to measure wall temperatures to better than 0.1°C. and to utilize small radial heat fluxes so as to minimize effects of natural convection.

A series of experiments was carried out on five different types of prescribed wall heat flux with water flowing in a vertical tube. The measured wall temperatures in these experiments compared well with those predicted by the theoretical solution. Small deviations which did exist were explained in terms of experimental errors.

The agreement found indicates that the analytical method may be safely used in the study of experimental data in such heat transfer and analogous mass transfer problems.

Ever since the classical work of Graetz in 1883 (2) there has been considerable interest in the problem of laminar-flow heat transfer. Many analytical solutions are available for the prediction of temperature in laminar-flow fields for various geometries and boundary conditions and with certain simplifying assumptions regarding fluid properties. Recently solutions have appeared for the case in which the flowing fluid is heated in a tube in which the wall heat flux varies in the direction of flow. Solutions of this type are useful not only in the heat transfer con-

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text, but also in connection with analogous problems of mass transfer.

In general, the analytical solutions have not been tested by experiment. Although some experimental data are available, the accuracy of these is not satisfactory. The purpose of this work was, therfore, to construct an experimental unit capable of producing accurate local temperature data, which might then be compared with theoretically predicted values. Since natural convection may have appreciable effects on heat transfer, the equipment was designed to minimize this effect. This apparently had not been done in previous experimental studies. An analysis similar to that proposed by Katz (*1*) for the related mass transfer problem was used to make the theoretical predictions.

Since the literature dealing with analytical solutions to problems of this type is rather extensive and has been well covered elsewhere (*3* to *6*), further complete review will not be attempted here. Suffice it to say that Siegel, Sparrow, and Hallman (*7*) and Katz (*1*) have recently presented pertinent solutions to the problem of laminar flow in a cylindrical tube with flux at the wall a prescribed function of axial distance.

Experimental work in the field of laminar-flow heat transfer is, unfortunately, almost nil. What little data are available were apparently obtained primarily to test empirical relationships for heat transfer coefficients. McAdams (*8*), for example, gives the results of such investigations in terms of the usual dimensionless correlations of Nusselt Number as a function of parameters such as Reynolds, Prandtl, and Grashof numbers. There appear to be very few data of the type desired here, that is, measurements of local heat fluxes and temperatures under laminar-flow conditions.

Recently Gross and Van Ness (*9*) did measure bulk and wall temperatures under constant heat-flux conditions for glycerol flowing in heated cylindrical tubes. Their work, however, was limited to the region of fully developed temperature and velocity profiles. No previous studies involving the use of an arbitrary, nonconstant wall heat flux have been reported.

Until the recent work of Rosen and Hanratty (*10*) and Scheele et al. (*11*), the effect of natural convection on laminar-flow heat transfer was generally neglected. These authors have shown, however, that natural convection can have serious effects on the shape of the velocity profile even at low rates of heat transfer. Such effects probably account for the 20 to 30% scatter frequently encountered in heat transfer data for the laminar region. Thus it is important that the heat flux in any experimental study be maintained low enough to avoid the complicating effects which natural convection can produce. On the basis of the results of Scheele (*11*), it appears that the normal laminar velocity profile will not be significantly changed by convective effects as long as the ratio of Grashof to Reynolds numbers is maintained less than 5.0.

THEORY

The analytical solution of the problem at hand has been well presented previously (*1, 7*). The main results of the solution shown by Katz (*1*) are given below, but the reader is referred to the original paper for details. The original problem was framed in terms of a mass transfer case with chemical reaction at the tube wall.

The basic energy equation for this problem may be written after the following assumptions have been noted:

1. Steady state conditions prevail.
2. Fluid properties are constant.
3. Laminar flow exists in the cylindrical tube.
4. Axial conduction in the flowing fluid is negligible compared with convective heat transfer.

The dimensionless energy equation is then

$$4(1-x^2)\frac{\partial w}{\partial x} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial w}{\partial x}\right) \quad (1)$$

The appropriate auxiliary conditions follow from the requirements that the inlet-fluid temperature be uniform, that there be symmetry about the tube axis, and that the conductive flux at the tube wall be everywhere equal to the prescribed function of axial position, $q(z)$. Then in dimensionless form

$$w = 1, \quad \theta \leq 0, \quad 0 < x < 1 \quad (2a)$$

$$\frac{\partial w}{\partial x} = 0, \quad x = 0, \quad \theta > 0 \quad (2b)$$

$$\frac{\partial w}{\partial x} + f(\theta) = 0, \quad x = 1, \quad \theta > 0 \quad (2c)$$

The problem posed by Equations (1) and (2) may be solved by methods involving use of the Laplace transformation (*1*). The final solution is given by the convolution integral shown in Equation (3):

$$w(x,\theta) = 1 + \int_o^\theta \left\{ 1 + \sum_{n=1}^{\infty} (\lambda_n a_n + b_n)\, \phi_n(x)\, e^{-\lambda_n(\theta-\tau)} \right\} f(\tau)\,d\tau \quad (3)$$

In this result ϕ_n, λ_n, and a_n and b_n represent the eigenfunctions, eigenvalues, and Fourier coefficients, respectively, of the eigenvalue problem associated with the system equations (*1*). These quantities, which are necessary for the use of Equation (3), have been determined by numerical procedures and have been reported elsewhere (*12*).

Once the variation of heat flux at the tube wall has been specified as a function of tube length [that is, $f(\theta)$ is known], Equation (3) may be used to calculate the temperature of the fluid at any point in the tubes. The same technique may be applied to problems in which the wall flux is a general function of wall temperature, since $q(t)$ at $r = R$ is $q[T(z, R)] = q(z)$ (a function of z only). Although such considerations may not be of importance in heat transfer, the opposite is true in mass transfer problems involving the flow of reactant fluid through a tube with a reactive surface. At such surfaces the rate of reaction or flux may depend in a general way on the concentration at the wall. Katz (*1*) has discussed this problem in detail.

The objective of the work to be described below was to carry out experiments in which the heat-flux functionality would be known and the wall temperatures measured. With this information, measured temperatures could be compared with those predicted by Equation (3) and the analytical method thus tested.

EXPERIMENTAL

The experimental equipment was designed to conform to the assumptions made in the analysis discussed above. The principal piece of equipment was a heat transfer tube, constructed to allow the heat input to be varied in the axial direction. Thermocouples were located along the length of the tube to measure the wall temperature. Additional pieces of equipment and instruments used in this system served to move the fluid through the heat transfer section and to measure the necessary operating parameters. The schematic flow system used is shown in Figure 1.

As shown in the figure, the fluid studied, which was water, was contained in a large, constant-temperature bath, in which the temperature could be controlled to ± 0.02°C. The capac-

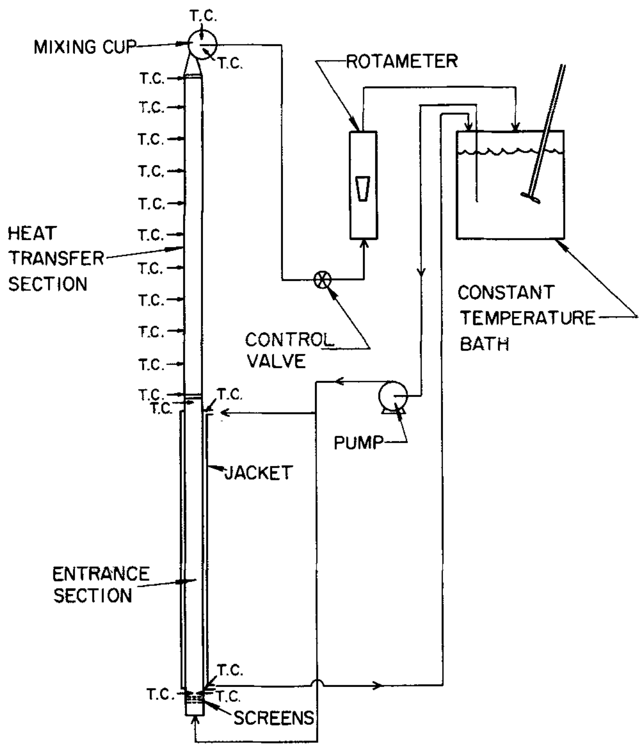

Fig. 1. Flow diagram of equipment.

ity of this bath was approximately 5 cu. ft. The fluid in the bath was also used as jacketing fluid in the entrance section. A fraction of the water was pumped from the bath through the entrance section to the heat transfer section, from which it flowed first into a mixing cup, then through a calibrated rotameter where the flow was measured, and finally back into the constant-temperature bath. An additional stream was taken from the constant-temperature bath and circulated through a concentric jacket located about the entrance section. The entrance and heat transfer sections were made from the same piece of copper pipe, and each section was 10 ft. long. They were positioned vertically one above the other, as shown in Figure 1. The copper pipe used was commercial, extra strong, ½-in. pipe, 0.843 in. O.D., and 0.555 in. I.D. The entrance section had an equivalent length of approximately 220 diam. which was felt to be sufficiently long to assure establishment of a fully developed laminar profile. The entrance section was jacketed with 1½-in. copper pipe for a distance of 9½ ft. The entire section and jacket were insulated with 1-in.-thick glass wool.

As stated above, the heat transfer section was located above the entrance section. However, in order to avoid the problem of heat conduction between the two sections, they were separated by a small nylon ring, which was carefully machined to fit snugly between them. The heat transfer section was located directly above this bottom separator. A similarly constructed separator was used at the top of the heat transfer section to join it to, and thermally to insulate it from, the mixing cup. Figure 2 shows the configuration of these sections.

The actual heat transfer section was made of 10 ft. of the same type of copper tube as that used for the entrance section. The outside surface of the heat transfer section was insulated with Teflon-coated fiber glass tape. An electric heating element was wound around the outside of this insulating tape and was then covered with additional tape. A 1-in. glass wool insulation layer was placed on top of the insulating tape to minimize heat losses from the heat transfer section and the heating coils. In addition, a 6-in.–diam. chimney was also placed about the entire heat transfer section. Air at a temperature close to that of the water in the tube was circulated through the chimney, thus further reducing losses.

The heat transfer section was heated by the wound resistance wire, of copper-nickel alloy, with a resistance of 0.366 ohm/ft. The wire was carefully and accurately wound on the tube by means of a lathe. A winding of ten coils an inch was used, and the subsequent total resistance of the wire on the tube was 98 ohms. Electrical connections were made to the coil at the ends, ½ ft. from each end of the tube, and at 1-ft. intervals in between—a total of twelve connections. A variable transformer used to supply the desired a.-c. voltage across the two ends of the heating coil was supplied with constant 115-volt alternating current by a voltage stabilizing transformer. Eleven variable resistors were placed in parallel with the heating coil in such a way that each might be connected across any of the sections of the heating coil. This was done so that the power to each 1-ft. section (or ½-ft. end section) of the coil might be varied simply by varying the effective resistance of that section. The resistors used ranged from 20 to 400 ohms and had a power rating of 4 watts. The voltage and current to the total heating coil were measured by appropriate meters, as were the voltage and resistance across each heat transfer section.

The mixing cup located on top of the heat transfer section served to mix completely the fluid flowing out of that section, so that its bulk temperature might be measured. The mixing cup consisted of a small cylindrical cup with a tangential entrance, which produced violent mixing of the fluid. Two movable thermocouples were located in this device. Readings from these thermocouples were taken to represent the bulk flow temperature of the fluid. It was found that the mixing in the cup was sufficient so that no differences could be found between the two thermocouples when they were moved in and out to various positions within the cup. The entire device was made of stainless steel and was enclosed in a 6-in. cubical box filled with glass wool insulation.

In order to measure the temperature of the heat transfer section wall, 1/16-in.–diam. holes were drilled through half the wall thickness of the heat transfer pipe at 1-ft. intervals. Thermocouples made from solid copper and constantan wire with polyvinyl insulation were inserted into these holes. The thermocouples were calibrated against one another and then against a resistance thermometer which was calibrated by the National Bureau of Standards. It is estimated that the standard temperature was known within ± 0.001°C. The electromotive forces from the various thermocouples located in the equipment were measured with a double potentiometer in conjunction with a galvanometer. It was possible to measure temperatures to 0.01°C.

This equipment was found to perform satisfactorily for the heat transfer studies that were made. The temperatures in-

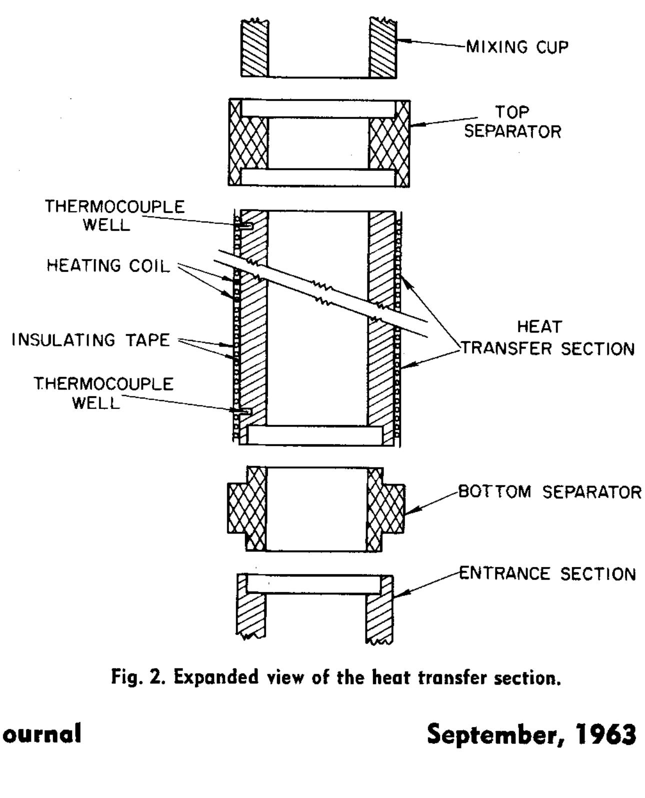

Fig. 2. Expanded view of the heat transfer section.

vestigated were approximately 30°C., and the equipment would reach steady state at any desired operating condition within approximately 30 min., whereupon steady and reproducible measurements were possible. The heat input to the fluid in all runs was extremely low, total heat input being maintained at 15 cal./sec. (63 watts) for the entire length of the tube.

RESULTS

The operation of the equipment was tested in a number of preliminary experiments. Then five series of experiments, designated runs 1 to 5, were performed as described below.

Run 1 was made with the heat flux constant over the entire length of heat transfer section. For this run, the driving voltage was connected directly across the entire heating coil with none of the variable resistors attached. This configuration allows for equal current flow, and hence equal power dissipation, over the entire length of the tube. In this run, as in all others, the water flowing into the heat transfer section was at a temperature quite close to that of the surroundings in order to minimize heat losses. The wall temperatures were measured at the eleven axial positions for a sequence of flow rates. The temperature of the fluid was also measured just before it entered the heat transfer section and in the mixing cup, and these values were then used to calculate the total energy transferred to the fluid as it passed through the heat transfer section.

Run 2 was made with the axial resistance of the heat transfer coil adjusted so as to provide for a linear heat input starting from zero at the bottom of the heat transfer section and increasing linearly with distance. The various resistances were adjusted so that the total heat generation rate was about the same as used in run 1; that is, approximately 15 cal./sec.

Run 3 was made with the heat flux varying axially from some finite value at the beginning of the tube to zero at the outlet end; that is, heat flux was just the opposite of that used in run 2.

Run 4 again used a linearly increasing heat-flux profile, but the heat flux in the initial section instead of zero was some finite value.

Run 5 was the reverse of run 4; that is, heat flux decreased linearly in the direction of flow from an initial high value to some lower, but nonzero, value at the outlet end. Equations which define these fluxes are given in Table 1.

These four variable heat-flux profiles were chosen because they were simple to set up experimentally and to analyze and yet gave four different types of curves in the wall temperature as a function of θ. It should be noted, of course, that because of the construction of the heating coil, the actual heat-flux profile achieved in each case was not a smooth linear profile, but rather a step-by-step approximation to such a profile. In view of the rather large number of sections that were used (ten), however, and the tendency of the thick copper tube to smooth out discontinuities in temperature, it is felt that the step-by-step nature may be reasonably neglected.

The resistance of each section of the heat transfer coil was set in advance at some desired value to produce the appropriate type of profile. The actual amount of heat transferred to the fluid was not, however, calculated from the power dissipation in the heating coil, but rather from an energy balance made on the flowing fluid. This was done in order to avoid the effect of heat losses from the coil to the outside surroundings. Although considerable insulation was used in this equipment and precautions were taken to ensure that no large temperature gradients existed between the heat transfer tube and the surrounding air temperature, nonetheless, because of the small fluxes, heat losses as high as 10% were sometimes encountered. For the purposes of calculation, it was assumed that the heat losses did not distort the shape of the heat-flux profile.

TABLE 1. EXPERIMENTAL HEAT-FLUX PROFILES

Run	Dimensionless profile
1	$f(\theta) = 0.1625$
2	$f(\theta) = 0.104\, V_{max}\theta$
3	$f(\theta) = 0.345 - 0.102\, V_{max}\theta$
4	$f(\theta) = 0.081 + 0.0330\, V_{max}\theta$
5	$f(\theta) = 0.202 - 0.0349\, V_{max}\theta$

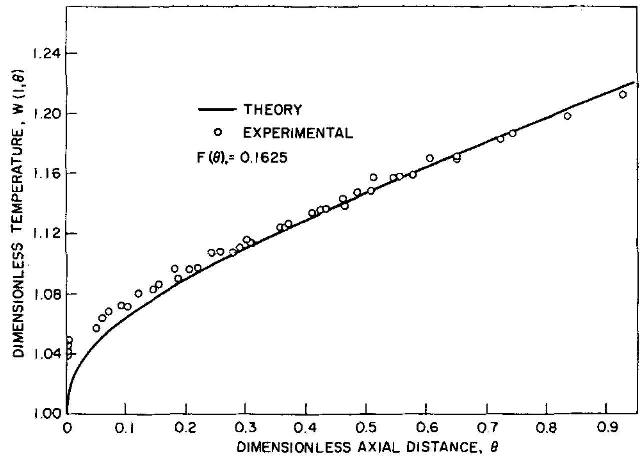

Fig. 3. Axial wall-temperature profile for constant heat flux.

Values of wall temperature were not directly determined in these experiments. Rather the dimensionless temperature defined previously was calculated directly from the various thermocouple electromotive force readings. These values for the dimensionless wall temperatures are shown on Figures 3 to 7 for each type of heat-flux profile. Here dimensionless temperature is plotted against dimensionless axial distance, θ. For conditions of constant wall flux all the data may be described by a single line, since in this case θ depends only on axial distance. However, for the other four runs, in which the profile varied with axial distance, the value of θ was a function both of axial position in the heat transfer tube and of fluid velocity. Hence it was necessary to make a sequence of experiments in which the fluid velocity was kept constant. Several such curves are shown on each of the figures corresponding to runs 2 to 5.

The analytical solution discussed earlier was then used, along with the actual heat-flux profile for each run, to calculate predicted values of dimensionless wall temperature as a function of θ. For these calculations physical properties of water were taken from the International Critical Tables. The number of terms required to give sufficient accuracy in the summation for the function $w\ (x, \theta)$ was found to be less than twenty in all cases of interest here. The predicted curves for each of the axial profiles used are also shown on Figures 3 to 7 as solid lines.

In all these experiments the Reynolds Number ranged between 350 and 640. This is sufficiently low to ensure a well-developed laminar profile in the absence of any disturbances in the flow path. Since such disturbances were absent from the smooth heat transfer tubes, and since the ratio of the Grashof Number to the Reynolds Number was maintained at less than five for the reasons discussed above, it was assumed in these experiments that the true parabolic laminar profile was actually obtained experimentally. In this connection it might be mentioned that the temperatures measured during the experiment were very steady, both at the walls of the tube and in the mixing cup, thus indicating the absence of any turbulence. On the contrary, when several preliminary runs were made at

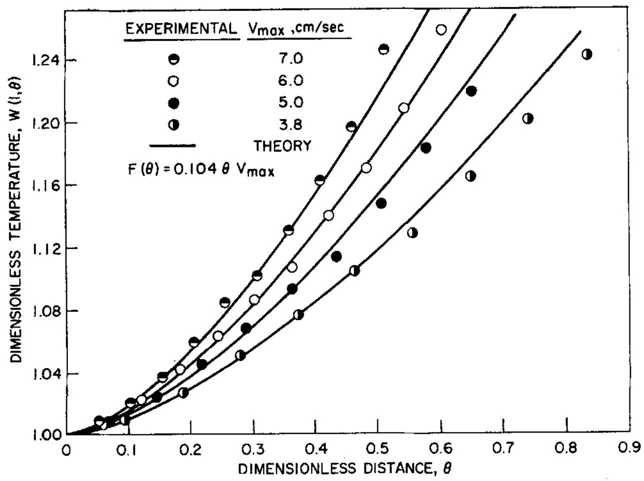

Fig. 4. Axial wall-temperature profiles for the case where the heat flux increases linearly from zero.

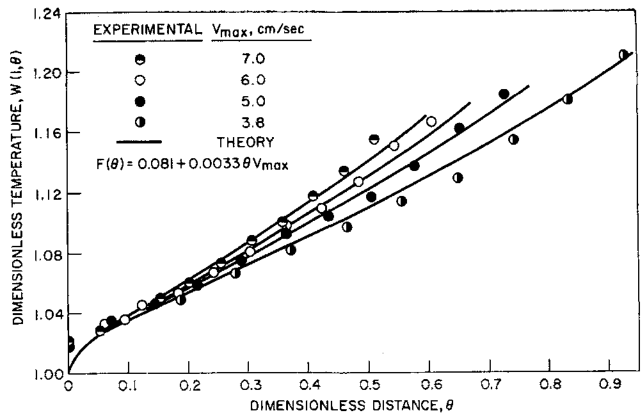

Fig. 6. Axial wall-temperature profiles for the case where the heat flux increases linearly from a nonzero value.

Reynolds Numbers corresponding to 2,000 to 2,300, the usual transition region, it was found that erratic and wildly fluctuating temperatures (± 1°) were produced in the mixing cup and along the wall. This, it appears, is evidence of the instability of the transition type of flow. The flow rate used in these experiments corresponded to 0 to 35 g. of water/sec. This value was very easily controlled with an accuracy of about ± 0.5 g./sec. over the entire range of operation.

DISCUSSION

As may be seen from Figures 3 to 7, the agreement between experimental dimensionless wall temperature and that predicted by Equation (3) was in general quite good. It should be borne in mind, when these figures are considered, that on the scale used a change of one unit in dimensionless temperature corresponds to a change of approximately 30°C. Therefore the largest discrepancy between predicted and calculated results, aside from that at values of $\theta = 0$, is about 0.2° to 0.3°C. It seems fair to say that these experiments have demonstrated that the theoretical and experimental data are in close agreement.

In that connection, it should be noted that these experiments are, in general, subject to fairly large errors because of the very low flow rates and heat fluxes used and the extremely small temperature differences which were encountered. The use of these conditions, however, was required in order to avoid the effects of natural convection. The absolute measurement of temperature was accurate to ± 0.12°C. (± 5 microvolts), although the internal consistency from thermocouple to thermocouple was considerably better, that is ± 0.01°C. However, taking the absolute temperature error along with the possible error in the flow rates, this could amount to a maximum error in calculated heat input of ± 2 cal./sec. This is to be compared with total heat input of approximately 14 to 15 cal./sec. (In practice, the actual error seemed to be closer to ± 1 cal./sec.) This still represents a fairly substantial error, but one which could not be easily diminished further. It should also be noted that the maximum error involved in the dimensionless wall temperatures reported on Figures 3 to 7 was 0.008 unit. The consistency of the data appears to be better than this, that is about 0.005 unit, which is approximately the diameter of the circles shown on these figures.

It can be seen that the major discrepancy between the theoretical and experimental data occurs in those regions where a large temperature difference existed between the fluid and either the surroundings or the heating coil. The latter condition occurs at the entrance of the tube in those runs where there is some initial heat input at that point, namely, runs 1, 3, 4, and 5. What apparently happens in this case is that heat is transferred down the thermocouple wire from the hotter heating coil to the wall of the tube, thus heating the junction of the thermocouple and giving a reading that is slightly higher than it should be. This particular problem was caused by the method in which the thermocouples were installed in the wall.

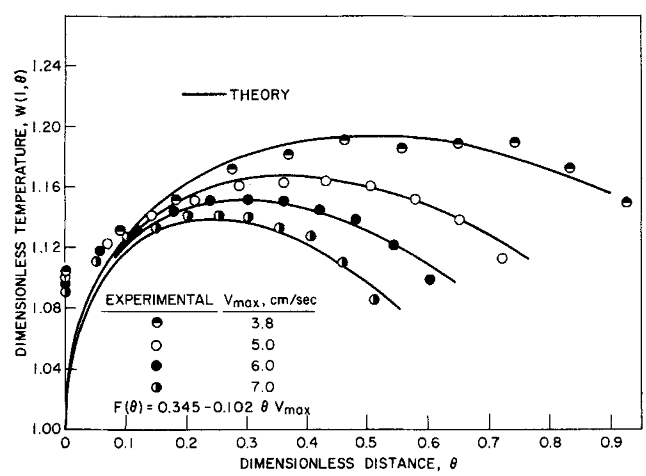

Fig. 5. Axial wall-temperature profiles for the case where the heat flux decreases linearly to zero.

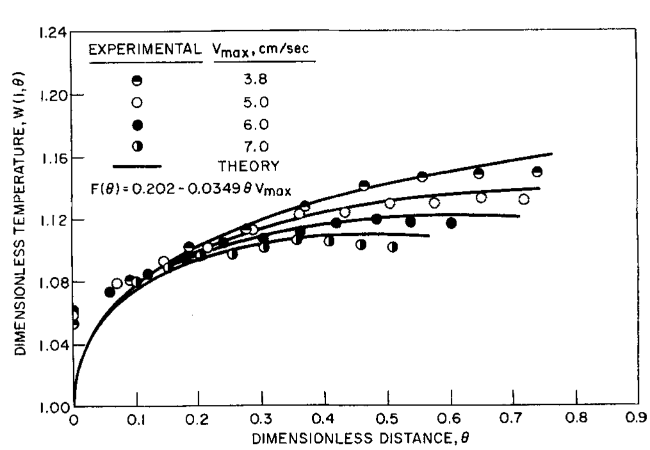

Fig. 7. Axial wall-temperature profiles for the case where the heat flux decreases linearly to a nonzero value.

The distance between the thermocouple junction and the heating coil was about 1/8 in. Hence, in those situations in which the wall temperature was significantly lower than the heating-coil temperature, a gradient existed in the thermocouple wire, which could produce additional heat transfer to the thermocouple tip. This situation might have been avoided if the thermocouples had been installed in a small trench around the outer circumference of the tube for a distance of perhaps 3/4 to 1 in. This would decrease the gradient in the actual thermocouple wire and eliminate most of the heat transfer to the thermocouple junction.

Further differences between experimental and calculated temperatures were found in those regions where the wall temperature was higher or lower than the surrounding temperature. These errors apparently were due to heat transfer between the wall and the surroundings. It was originally thought that the use of the insulation discussed above would have been sufficient to eliminate such heat transfer; however, this apparently was not the case. Since the heat fluxes involved were extremely small, small variations due to transfer with the surroundings were certainly not unreasonable. Further, if the heat transfer to the surroundings was highest at those places where the difference in temperature was the highest, as one would have expected, then the actual heat flux into the fluid at such points would have been lower than calculated on the basis of the assumed heat-flux profile. Such errors would result in the theoretical calculation yielding somewhat higher temperatures than were measured experimentally. This situation was found at the hotter end of the tube. The effect might be decreased by use of additional insulation.

It should perhaps be mentioned at this point that the location of the thermocouple junction approximately half way through the thickness of the copper tube should not cause these measured temperatures to differ significantly from the temperatures at the liquid-tube interface. This is so because of the very small heat fluxes which were used and the resultant small gradients which would exist in the copper tube as determined from calculations using the standard conductivity of copper. Thus, as far as radial heat transfer through the copper tube was concerned, the conductivity was such that the tube was essentially a perfect conductor, and negligible gradients existed. On the other hand, examination of the data shows that there was, of course, a temperature gradient in the axial direction in every case, which would produce some heat transfer axially through the walls of the tube itself, thus tending to smear out the calculated heat flux profiles assumed in this work. Although this effect was undoubtedly present, it was neglected in this work because the axial gradients are rather small and the area available for heat transfer is also small in this direction. Calculations indicate that the ratio of heat transferred radially to that transferred axially within the copper tubing was at least of order 10 and in many cases was no doubt higher.

It seems reasonable therefore to state that this study has proved the validity of the analytical treatment of the problem of laminar-flow heat transfer with arbitrary heat-flux profiles. The experimental data are in very close agreement with the predicted equations, although the data were obtained in a region of low heat fluxes and low Reynolds numbers. Further work to establish more firmly the allowable limits of this region would, of course, be valuable.

As stated earlier, the mathematical approach used was originally developed for an analogous problem of laminar-flow mass diffusion with chemical reaction at a tube wall. In view of the success of the present work, it seems reasonable to assume that similar success might be expected in the experimental mass transfer situation. Certain obvious differences exist in the two situations, of course, and appropriate care is necessary to ensure that the assumptions which were used in the heat transfer problem apply equally well in the mass transfer case. However, if such precautions are taken, experiments in a laminar-flow reactor would be useful in obtaining relationships between the mass flux at the wall of a tube and the local wall concentration without the need for measurements made near the surface. Such information, as pointed out by Katz, would be of extreme value in a study of the kinetics of solid catalyzed reactions. No such experiments have yet been reported in the literature.

CONCLUSIONS

On the basis of the results described above, it may be concluded that

1. The basic mathematical analysis is valid and presents a practical method of calculation for the range of variables tested. Therefore, this analysis may be useful in solving similar problems with nonlinear-flux boundary conditions arising in heat and mass transfer.
2. In view of the good agreement between experimental and predicted results, the experimental design used here is adequate for accurate measurements of the wall temperature and the heat flux, although refinements are possible which would increase the accuracy of the data.

NOTATION

a_n, b_n = Fourier coefficients
C_p = heat capacity, cal./(g.) (°C.)
$f(\theta)$ = dimensionless heat flux, $\dfrac{R\,q(z)}{T_o\,k}$
k = thermal conductivity, cal./(sec.) (cm.) (°C.)
$q(z)$ = heat flux, a function of z, cal./(sec.) (sq.cm.)
r = radial coordinate, cm.
R = tube radius, cm.
T = temperature, °C.
T_o = fluid inlet temperature, °C.
V_{max} = maximum (center line) fluid velocity, cm./sec.
$w(x,\theta)$ = dimensionless temperature = T/T_o
x = dimensionless radial distance = r/R
z = axial position variable, cm.

Greek Letters

α = thermal diffusivity = $\dfrac{k}{\rho\, C_p}$, sq.cm./sec.
θ = dimensionless axial position = $4\,\alpha\, z/R^2\, V_{max}$
λn = eigenvalue
ρ = fluid density, g./cc.
ϕ_n = eigenfunction

LITERATURE CITED

1. Katz, Stanley, *Chem. Eng. Sci.*, **10**, 202 (1959).
2. Graetz, L., *Ann. Physik*, **18**, 79 (1883).
3. Brown, G. M., *A.I.Ch.E. Journal* **6**, 179 (1960).
4. Singh, S. M., *Appl. Sci. Res.*, **7A**, 325 (1958).
5. Sellars, J. B., Myron Tribus, and J. S. Klein, *Trans. Am. Soc. Mech. Engrs.*, **78**, 441 (1956).
6. Reynolds, W. C., *ibid.*, **82C**, 1 (1959).
7. Siegel, R., E. M. Sparrow, and T. M. Hallman, *Appl. Sci. Res.*, **7A**, 386 (1958)
8. McAdams, W. M., "Heat Transmission," 3 ed., p. 229, McGraw-Hill, New York (1959).
9. Gross, J. F., and H. C. Van Ness, *A.I.Ch.E. Journal*, **3**, 172 (1957).
10. Rosen, E. M., and T. J. Hanratty, *ibid.*, **7**, 112 (1961).
11. Scheele, G. F., E. M. Rosen, and T. J Hanratty, *Can J. Chem. Eng.*, **38**, 67 (1960).
12. Dranoff, J. S., *Math. of Computation*, **15**, 403 (1961).

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